Pack 11: Division

Session A: Division and multiplication

Resources needed: Counters (or applicable alternative)

The purpose of this session is to revise understanding of the connection between multiplication and division and use knowledge of multiples to make sense of division calculations.

Talk Task
An array of counters each labelled with the value of 10 is presented. Ask pupils to tell you what they can see and describe any calculations they think the model represents. This should be a familiar model of multiplication and pupils should be able to identify and describe equal groups and write multiplication calculations.

Use the suggested sentences to discuss the language ‘multiple of’ and ‘divisible by’. Connect these ideas using the array of counters to support explanations.

• 210 is a multiple of 70
• 210 is divisible by 70. I can divide 210 into 3 groups of 70
• 210 is a multiple of 3
• 210 is divisible by 3. I can divide 210 into 3 equal groups. Each group is 70.

Think carefully about how to describe the division based on the model. For example, it is not easy to see 210 divided into 70 groups of 3 with this model.

• 210 is a multiple of 30
• 210 is divisible by 30. I can divide 210 into 7 groups of 30.
• 210 is a multiple of 7
• 210 is divisible by 7. I can divide 210 into 7 equal groups. Each group is 30.

While working through these, write division calculations that the array can represent. Pause and reflect on the many calculations that can be worked out.

For the next section, build on the derived facts identified so far to identify numbers that are divisible by 7. A number line is provided that restricts the task to numbers between 130 and 230. For each number identified, discuss how you know and how to record as a division calculation and a multiplication calculation.

Activity
This activity guides students through a similar experience of deriving and recording facts. Then a multiplication fact is given and pupils are to use this to solve word problems involving division and related facts. Extend this task by asking pupils to generate their own word problems.
1) Copy and complete the calculations this array could represent as the value of each counter is changed.
   a) Each counter has a value of 1
      
      \[
      \begin{align*}
      3 \times 8 &= 24 \\
      8 \times 3 &= 24 \\
      24 \div 8 &= 3 \\
      24 \div 3 &= 8 \\
      \end{align*}
      \]

   b) Each counter has a value of 10
      
      \[
      \begin{align*}
      30 \times 8 &= 240 \\
      8 \times 30 &= 240 \\
      240 \div 8 &= 30 \\
      240 \div 30 &= 8 \\
      3 \times 80 &= 240 \\
      80 \times 3 &= 240 \\
      240 \div 80 &= 3 \\
      240 \div 24 &= 10 \\
      \end{align*}
      \]

2) Use the fact that \(4 \times 6 = 24\) to answer the following:

<table>
<thead>
<tr>
<th><strong>£240 is shared equally between 4 people. How much does each person get?</strong></th>
<th><strong>240 grams of sugar is split into bowls with 60 g in each. How many bowls of sugar are there?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{£60})</td>
<td>4 bowls of sugar</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Completing a level of a game gets you 60 points. You have 2400 points. How many levels have you completed?</strong></th>
<th><strong>I do 40 minutes of exercise every day. How many days until I have done 240 minutes?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>40 levels</td>
<td>6 days</td>
</tr>
</tbody>
</table>
### Pack 11: Division

**Session B: Halving strategies**

**Resources needed:** Dienes

The purpose of this session is to explore different division strategies that involve halving. Exploring different ways to complete the same calculation allows you to discuss which strategy you would choose.

**Talk Task**

Four models are shown to represent different ways of seeing half of 72. Start by asking pupils to describe what they can see and discuss what is the same and what is different. Encourage pupils to build and label each model.

Below the models are four sets of calculations describing the steps of four different strategies. Read and discuss which matches which model and think about why and how you can see each calculation in the arrangement of the blocks. Take the time to attach each step of the calculation to the model that it matches, describing the role of each number.

The top two strategies partition 72 and halve each part. On the left, 72 is partitioned into 70 and 2 and on the right into 60 and 12. It is important to encourage pupils to be flexible in their choices of how to partition and consider options other than tens and ones.

The bottom two strategies identify a multiplication calculation and halve one of the factors to halve the product.

Having discussed each model and strategy, talk about which you would choose to use for this calculation and think about other calculations that suit each strategy. Extend this task by exploring a similar calculation such as 58 ÷ 2 or 96 ÷ 2 and building or drawing models that show different ways this could be calculated. The focus should be on explaining and seeing the structure.

### Activity

The activity sheet uses arrays of counters to illustrate division strategies involving halving and repeated halving. Pupils are to look at each step and complete the model by drawing rings around sections of the array and complete the empty boxes.

Extend this task by challenging students to find other calculations that suit each of the strategies explored as well as calculation that do not.
Activity: Halving strategies

1) The images show a halving strategy. Complete the boxes.

Two groups of 18

36 \div 2 = 18

Four groups of 9

36 \div 4 = 9

2) Complete the images to match the steps of the halving strategy.

Half of 24 is 12

24 \div 2 = 12

Half of 12 is 6

24 \div 4 = 6

Half of 6 is 3

24 \div 8 = 3

3) Complete the strategy and show it works with another calculation.

To divide a number by 6, I can halve and then divide by 3

Half of 48 is 24

24 \div 2 = 12

Half of 186 is 96

96 \div 2 = 48

24 divide by 3 is 8

48 \div 3 = 16

96 divide by 3 is 32

48 \div 6 = 8

186 \div 6 = 32

There are different ways to complete question 2 and many different examples that could be given for question 3.
### Pack 11: Division

#### Session C: Division structures

**Resources needed:** A way to draw number lines

The purpose of this session is to explore different ways of interpreting the same division calculation. Division as sharing and division as grouping. For example, \(10 \div 2\) can mean 10 shared between 2 or it can mean 10 put into groups of 2. Number lines are used as the model for exploring this.

**Talk Task**

Division calculations can be interpreted in two different ways and these different ways of seeing division can lead to different calculation strategies. This task uses number lines as the model to explore the same calculation in two different situations.

Read what each person says and think about what is the same and what is different between the two situations.

- *In both we know the total number and it is the same, 150*
- *We know the number of pencils in each pack, 30. We don’t know the number of packs.*
- *We don’t know the number of pens in each pack. We know the number of packs, 30.*

Ask pupils to explain why the same calculation, \(150 \div 30\) can be used to solve both situations. Highlight that this calculation can mean ‘how many 30s in 150?’ or it can mean ‘150 is split into 30 equal groups, how many in each group?’

Look at the number line for each situation and spend time making sense of the information displayed. Ask pupils to place more information onto each line.

*One pack is 30 pencils, how many packs of 30 is 150? Two packs is 60, four packs is 120 and five packs is 150 pencils.*

*150 pens is the amount in 30 packs, what is 150 divided into 30 equal parts? 10 packs is 150 divided into 3 which is 50. If 10 packs is 50 then 1 pack has 5 pens.*

**Activity**

The activity sheet uses the context of frogs jumping to provide further practice with using a number line. Extend the activity by asking pupils to record multiplication and division calculations that each question is connected with. They can also generate their own situations to show the two interpretations of division.

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Activity: Division structures

1) A frog travels 8cm for each jump.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>16</td>
<td>40</td>
<td>64</td>
<td>80</td>
<td>120</td>
</tr>
</tbody>
</table>

- a) How far has it travelled after 2 jumps? 16 cm
- b) How many jumps does it take to travel 40 cm? 5
- c) How many jumps does it take to travel 64 cm? 8
- d) How far has it travelled after 10 jumps? 80 cm
- e) How many jumps does it take to travel 120 cm? 15

2) This frog has jumped 15 equal jumps and travelled 75 cm.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>15</td>
<td>25</td>
<td>50</td>
<td>75</td>
</tr>
</tbody>
</table>

- a) How far has it travelled after 5 jumps? 25 cm
- b) How far has it travelled after 10 jumps? 50 cm
- c) How big is each jump? 5 cm
- d) How far has it travelled after 3 jumps? 15 cm
### Pack 11: Division

### Session D: Models of division

**Resources needed:** Dienes

The purpose of this session is to explore different models for representing division. All of the models have been used in previous packs to represent multiplication showing that knowledge of multiples can be used to divide.

**Talk Task**

Three different models are provided with some labels. Start by asking pupils to describe what they see, encouraging them to label the models and think about calculations they could represent.

Three division calculations are given. Take time to match each calculation to a representations prompting pupils to give clear explanations and show why.

The array of Dienes shows 9 tens and 3 ones which is 93. This has been divided into 3 equal rows and each row has length 31. This can represent the multiplication calculation \(31 \times 3 = 93\) and so it also represents the division calculation \(93 \div 3 = 31\).

The rectangle is labelled with an area of 124. One side has length 4 and this information connects it to the division calculation \(124 \div 4\). Take time to discuss why the other length must be 30 and 1.

The number line shows one part with a value of 3, 30 parts with a value of 90, and asks what number of parts have a value of 96. It is asking how many threes are there in 96. This is the division calculation \(96 \div 3 = 32\).

Having discussed each of the models and the calculations they represent, discuss what is the same, what is different and what is the relationship between these calculations. You can extend the session by creating the other models for each calculation and by varying the calculations in other ways and representing those.

**Activity**

The activity sheet provides similar experiences of completing calculations, labelling models and using understanding of multiplication to divide. The final question asks pupils to create a representation for a given calculation. There are lots of possible ways to do this and you can encourage pupils to create more than one representation.
Pack 11 Session D
Activity: Models of division

1) Label the models and complete the calculations.

\[
\begin{align*}
92 \div 4 &= 23 \\
23 \times 4 &= 92 \\
162 \div 6 &= 27 \\
27 \times 4 &= 162
\end{align*}
\]

2) Complete the calculations and label the number line.

a) \(4 \times 6 = 24\) \(24 \div 6 = 4\)

b) \(30 \times 6 = 180\) \(180 \div 6 = 30\)

c) \(34 \times 6 = 204\) \(204 \div 6 = 34\)

3) Draw a model to represent \(72 \div 3 = 23\).

There are lots of ways to complete this. Look for models that show 72 as 3 groups of 23 or as 23 groups of 3.